MPRI - Cours 2-8: vérification des systèmes temps-réel

TD: timed automata

1 Region graph

For the timed automaton on Fig. 1, left, build its region automaton.

2 Bisimulation

Prove that there is no time-abstract bisimulation with finitely many equivalence classes on the state space of the LHA on Fig. 1, right.

3 Complementable or not?

A useful result

We recall that the following problem (Universality) is undecidable: Given a timed automaton $\mathcal{A}$, is it true that $L(\mathcal{A}) = \mathcal{U}$ with $\mathcal{U}$ set of all the timed words on Σ?

3.1 Folk theorem - S. Tripakis

Prove that there is no algorithm that given a timed automaton A

- answers YES or NO whether $\overline{L(A)}$ is timed regular;
- and if the answer is YES builds a timed automaton B such that $\overline{L(A)} = L(B)$.

Hint: suppose that such an algorithm exists and use it to decide Universality.

3.2 Undecidability of Complementability - O. Finkel

Prove that there is no algorithm that, given a timed automaton A, answers YES or NO whether $\overline{L(\mathcal{A})}$ is timed regular.

construction: We will again reduce Universality to Complementability. Let $\mathcal{A}$ be a timed automaton over Σ, $\mathcal{L} = L(\mathcal{A})$ its language; $\mathcal{M}$ your favorite non-complementable timed language, and c a new letter.

We construct a new timed language $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3$ with

$$\begin{aligned}
\mathcal{N}_1 &= \{\texttt{all the words with 0 or } \geq 2 \texttt{ letters } c\};\\
\mathcal{N}_2 &= \{utcv | u \in \mathcal{L}, t \in \mathbb{R}_+, v \in \mathcal{U}\} = \mathcal{L}\mathbb{R}_+c\mathcal{U};\\
\mathcal{N}_3 &= \{utcv | u \in \mathcal{U}, t \in \mathbb{R}+, v \in \mathcal{M}\} = \mathcal{U}\mathbb{R}_+c\mathcal{M}.
\end{aligned}$$

questions

1. Let $\mathcal{L} = \mathcal{U}$. Compute $\mathcal{N}$, is its complement timed regular?
2. Let now $\mathcal{L} \neq \mathcal{U}$. How to prove that the complement of $\mathcal{N}$ is not timed regular (it is not easy!).
3. Terminate the proof.

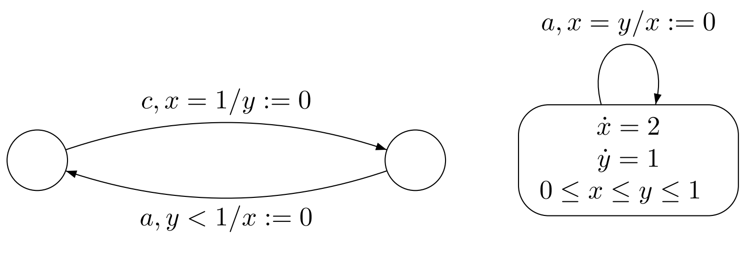

Figure 1: Left: timed automaton. Right: an LHA