MPRI - Cours 2-8: vérification des systèmes temps-réel

TD: decidability

Updatable Timed Automata

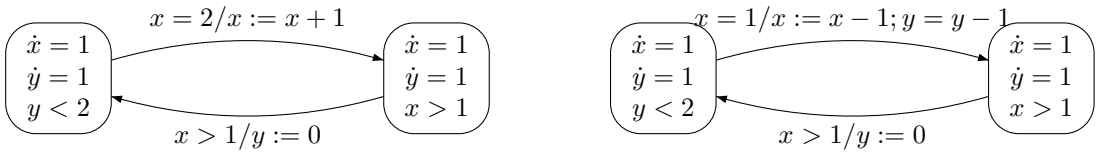

Figure 1: A U+ and a U- automata

Object of study. Two slight extensions of timed automata (due to Bouyer et al.) are considered. *U+ automata* are timed automata with the only difference that resets $x := x + 1$ are allowed. *U- automata* are timed automata with the only difference that resets $x := x - 1$ are allowed.

We are mainly interested in the decidability of the predicate $\mathcal{R}$, which is defined as follows: given an U+/U- $\mathcal{A}$ and two of its control locations p and q, the predicate $\mathcal{R}(\mathcal{A}, p, q)$ is true if and only if there exists a run of $\mathcal{A}$, starting at p with all the clocks at 0 and terminating at q with arbitrary values of clocks.

Question 1: semi-decidability.

- Prove that $\mathcal{R}$ is semi-decidable (recursively enumerable) for U+ and U-.

Question 2: decidability for U+.

- Prove that $\mathcal{R}$ is decidable for U+.

  Hint: Youn can transform a U+ into a normal TA, by replacing (simulating) each incrementation of x by a gadget TA. The main difficulty is not to destroy other clocks.

  Hint: Alternatively you can use a version of the region graph construction.

- Explain why your decision procedure does not extend to U-.

Question 3: undecidability for U-.

We suggest to encode a counter value n by a clock $x = n$.

- Give a black-box description (characterize the input-output relations) of gadget U- that you need in order to simulate one counter.
- Build these gadgets.

  Hint: If you are unable to, you can still proceed with the subsequent sub-questions.

- Give a black-box description (characterize the input-output relations) of gadget U- that you need in order to simulate two counters.
- Build these gadgets.

  Hint: If you are unable to, you can still proceed with the last sub-question.

- Terminate the proof of undecidability of $\mathcal{R}$ by simulation of a Minsky Machine.