**Updatable Timed Automata**

Object of study. Two slight extensions of timed automata (due to Bouyer et al.) are considered. *U+ automata* are timed automata with the only difference that resets $x := x + 1$ are allowed. *U- automata* are timed automata with the only difference that resets $x := x - 1$ are allowed.

We are mainly interested in the decidability of the predicate $R$, which is defined as follows: given an U+/U- $A$ and two of its control locations $p$ and $q$, the predicate $R(A, p, q)$ is true if and only if there exists a run of $A$, starting at $p$ with all the clocks at 0 and terminating at $q$ with arbitrary values of clocks.

**Question 1:** semi-decidability.
- Prove that $R$ is semi-decidable (recursively enumerable) for U+ and U-.

**Question 2:** decidability for U+.
- Prove that $R$ is decidable for U+.

  **Hint:** You can transform a U+ into a normal TA, by replacing (simulating) each incrementation of $x$ by a gadget TA. The main difficulty is not to destroy other clocks.

  **Hint:** Alternatively you can use a version of the region graph construction.

- Explain why your decision procedure does not extend to U-.

**Question 3:** undecidability for U-.

We suggest to encode a counter value $n$ by a clock $x = n$.

- Give a black-box description (characterize the input-output relations) of gadget U- that you need in order to simulate one counter.
- Build these gadgets.

  **Hint:** If you are unable to, you can still proceed with the subsequent sub-questions.

- Give a black-box description (characterize the input-output relations) of gadget U- that you need in order to simulate two counters.
- Build these gadgets.

  **Hint:** If you are unable to, you can still proceed with the last sub-question.

- Terminate the proof of undecidability of $R$ by simulation of a Minsky Machine.

- Does there exist a finite bisimulation on states of U- automata?