

2-34-1 : Quantum information and applications

Exercises for Part I

1. Suppose that a one qubit unitary is given as $U|0\rangle = z_1$ and $U|1\rangle = z_2$ with $z_0, z_1 \in \{0, 1\}$. Is this unitary for any values of z_1, z_2 ? Give a necessary and sufficient condition for a linear application given as a map from elements of the standard basis to elements of the standard basis to be unitary.
2. The Fredkin gate F is a classical controlled SWAP gate with three Boolean inputs (a, b, c) and three Boolean outputs (a', b', c') . The output is (a, c, b) if $a = 1$ and (a, b, c) otherwise.
 - (a) Give an explicit matrix for F and show that it is unitary.
 - (b) Give the outcome of this unitary on the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |0\rangle$
 - (c) Give a unitary for a quantum AND gate. Show how to use the unitary for F to compute a quantum gate for AND.
 - (d) Show how to simulate the Toffoli gate using three Fredkin gates.
3. Consider Bernstein Vazirani's algorithm. Run the algorithm on the input $x=0110$ and compute the outcome of the algorithm.
4. Compute the probability of success of Grover's algorithm after one iteration, when the input is 00001010.