Computing Models

Exercise 1 Let $\mathcal{G} = \{G_1, \cdots, G_\ell\}$ a set of digraphs with a common set of nodes V, and let $\langle G_1, \cdots, G_k\rangle$ the class of networks defined as

$$\langle G_1, \cdots, G_k\rangle = \{\mathbb{G} : \forall t \in \mathbb{N}^*, \exists k \in [\ell],\ \mathbb{G}(t) = G_k\}.$$

1. Show that this *class of networks generated by* $\{G_1, \cdots, G_k\}$ is closed.
2. Give a (more) general form of classes of networks that are all closed.
3. Give several classes of networks that are not closed.

Exercise 2

1. In what sense agents are actually supposed to start synchronously, i.e., they all start to run algorithms at the same round?
2. How to modify the (notion of solvability in the) model to take into account asynchronous starts?
   Indication: you may introduce the notion of a *starting schedule* for a set of agents V as a mapping $\mathbb{S} : V \to \mathbb{N}^*$, and then define the execution of an algorithm for a network $\mathbb{G}$ and a starting schedule $\mathbb{S}$ (with slight modifications/extensions of the sending and transition functions).
3. Why the above model extension does not work for self-stabilizing algorithms?

Exercise 3 According to the Heard-Of model, give the classes of networks that "naturally" correspond to reliable links and:

1. synchronous systems over a complete network and at most f faulty senders;
2. synchronous systems over a complete network and at most f crashes;
3. asynchronous systems over a complete network and at most f crashes;
4. asynchronous systems over a complete network and at most f initial crashes.

To what classes of networks does the Santoro and Widmayer's theorem apply?

Exercise 4

1. Give a bound on the diameter of a strongly connected digraph with n nodes.
2. Same question with a dynamic graph that has a finite diameter.
3. Show that the product of $n-1$ strongly connected digraphs with the same set of n nodes and a self-loop at each node is the fully-connected digraph.