1 Region graph

For the timed automaton on Fig. 1, left, build its region automaton.

2 Bisimulation

Prove that there is no time-abstract bisimulation with finitely many equivalence classes on the state space of the LHA on Fig. 1, right.

3 Complementable or not?

A useful result

We recall that the following problem (Universality) is undecidable: Given a timed automaton $A$, is it true that $L(A) = \mathcal{U}$ with $\mathcal{U}$ set of all the timed words on $\Sigma$?

3.1 Folk theorem - S. Tripakis

Prove that there is no algorithm that given a timed automaton $A$

- answers YES or NO whether $L(A)$ is timed regular;
- and if the answer is YES builds a timed automaton $B$ such that $L(A) = L(B)$.

Hint: suppose that such an algorithm exists and use it to decide Universality.

3.2 Undecidability of Complementability - O. Finkel

Prove that there is no algorithm that, given a timed automaton $A$, answers YES or NO whether $\overline{L(A)}$ is timed regular.

construction: We will again reduce Universality to Complementability. Let $A$ be a timed automaton over $\Sigma$, $L = L(A)$ its language; $M$ your favorite non-complementable timed language, and $c$ a new letter.

We construct a new timed language $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3$ with

$\mathcal{N}_1$ = \{all the words with 0 or $\geq 2$ letters $c$\};
$\mathcal{N}_2$ = \{utce|u $\in \mathcal{U}$, t $\in \mathbb{R}^+$, v $\in \mathcal{U}$\} = $\mathcal{L}\mathbb{R}_+d+t$;
$\mathcal{N}_3$ = \{utce|u $\in \mathcal{U}$, t $\in \mathbb{R}^+$, v $\in \mathcal{M}$\} = $\mathcal{U}\mathbb{R}_+c\mathcal{M}$.

questions

1. Let $\mathcal{L} = \mathcal{U}$. Compute $\mathcal{N}$, is its complement timed regular?
2. Let now $\mathcal{L} \neq \mathcal{U}$. How to prove that the complement of $\mathcal{N}$ is not timed regular (it is not easy!).
3. Terminate the proof.

\[c, x = 1/y := 0\]
\[a, y < 1/x := 0\]
\[\dot{x} = 2\]
\[\dot{y} = 1\]
\[0 \leq x \leq y \leq 1\]

Figure 1: Left: timed automaton. Right: an LHA