Updatable Timed Automata

Object of study. Two slight extensions of timed automata (due to Bouyer et al.) are considered. \( U^+ \) automata are timed automata with the only difference that resets \( x := x + 1 \) are allowed. \( U^- \) automata are timed automata with the only difference that resets \( x := x - 1 \) are allowed.

We are mainly interested in the decidability of the predicate \( \mathcal{R} \), which is defined as follows: given an \( U^+/U^- \mathcal{A} \) and two of its control locations \( p \) and \( q \), the predicate \( \mathcal{R}(\mathcal{A}, p, q) \) is true if and only if there exists a run of \( \mathcal{A} \), starting at \( p \) with all the clocks at 0 and terminating at \( q \) with arbitrary values of clocks.

Question 1: semi-decidability.

- Prove that \( \mathcal{R} \) is semi-decidable (recursively enumerable) for \( U^+ \) and \( U^- \).

Question 2: decidability for \( U^+ \).

- Prove that \( \mathcal{R} \) is decidable for \( U^+ \).
  
  \textbf{Hint:} You can transform a \( U^+ \) into a normal TA, by replacing (simulating) each incrementation of \( x \) by a gadget TA. The main difficulty is not to destroy other clocks.

  \textbf{Hint:} Alternatively you can use a version of the region graph construction.

- Explain why your decision procedure does not extend to \( U^- \).

Question 3: undecidability for \( U^- \).

We suggest to encode a counter value \( n \) by a clock \( x = n \).

- Give a black-box description (characterize the input-output relations) of gadget \( U^- \) that you need in order to simulate one counter.

- Build these gadgets.
  
  \textbf{Hint:} If you are unable to, you can still proceed with the subsequent sub-questions.

- Give a black-box description (characterize the input-output relations) of gadget \( U^- \) that you need in order to simulate two counters.

- Build these gadgets.
  
  \textbf{Hint:} If you are unable to, you can still proceed with the last sub-question.

- Terminate the proof of undecidability of \( \mathcal{R} \) by simulation of a Minsky Machine.