

MPRI 2-24-1: Algorithms and Uncertainty (2024)

Homework 5

Due on October 24, beginning of class

Instructions You can write your solutions either in English or French. Please observe the homework policy as described in the course web page.

Consider the online Steiner tree problem in the Random Order Model (ROM). That is, the adversary fixes a set S of requests (vertices of a known graph) which are then presented to the algorithm as a permutation of S , generated uniformly at random. Denote by n the number of terminals, i.e., $|S| = n$.

- (a) Use the property that duplicating requests does not change the cost of the Steiner tree, to show a lower bound of $\Omega(\log \log n)$ on the competitive ratio of any online algorithm under the ROM.
- (b) Now use a second property: that the worst-case instance for the (standard) online Steiner tree problem can be defined over the diamond graph, as we saw in class. Show how to obtain an improved lower bound of $\Omega(\frac{\log n}{\log \log n})$.

Hint: The solution is sketched in Section 5.1.1 of the survey by Gupta and Singla <https://arxiv.org/pdf/2002.12159>. You need to fill in the details, and make the arguments formal.

Warning: they may (or may not) exist some typos, so do not blindly copy the proposed approaches!