MPRI 2-24-1: Algorithms and Uncertainty (2024)

Homework 1 *Due on September 26, beginning of class*

Instructions You can write your solutions either in English or French. Please observe the homework policy as described in the course web page.

Problem 1 [5 marks]

Consider the standard paging problem with N different pages and a cache of size $k \leq N$. Let A be an algorithm which, upon a page fault, evicts a page from the cache that has been requested the least number of times so far, among all cache pages. Prove that A is not competitive, in that for any given c, we can show that A has competitive ratio at least c. *Hint:* Suffices to consider N that is only a little larger than k.

Problem 2 [5 marks]

Consider the paging problem in which the algorithm LRU has access to a cache equal to k, whereas the optimal offline algorithm can only use a cache of size $h \leq k$. Give an expression of the competitive ratio of LRU with respect to such an "inferior" optimal offline algorithm as a function of h, k. Explain the significance of your results for the cases $h = k$, and $h = k/2$, respectively.