Stochastic analysis of online algorithms for Steiner tree problems

Areas of interest: Analysis of algorithms, theoretical computer science, operations research.
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General presentation of the topic  In online computation, the input is not known to the algorithm in advance, but instead is revealed incrementally, as individual pieces of the input become available. For each new item, the online algorithm must perform an irrevocable decision. The objective is to design algorithms that perform efficiently (with respect to an “offline” optimal algorithm that has complete knowledge of the input). The standard approach in comparing the performance of online algorithms is by means of competitive analysis: here, we compare an online algorithm against the optimal offline algorithm on the worst-case input sequence. In the past decades, a large number of combinatorial optimization problems have been studied from the point of view of competitive analysis; see the textbook [1] for an in-depth treatment of this field.

Is this worst-case measure the best way to analyze online algorithms? It turns out that often requests are not adversarial, but are rather generated according to some distribution, which may or may not be known to the algorithm designer. Thus the question: can we improve upon the worst-case competitive ratio under such a stochastic setting of request generation? This is the topic of work by Garg et al. [2] who presented models that incorporate both stochastic and competitive analysis. For instance, under certain assumptions, [2] showed that the stochastic competitive ratio of online Steiner tree can be $O(1)$, whereas in worst case a competitive ratio of $\Omega(\log k)$ is unavoidable ($k$ is the number of terminals).

Objectives  The purpose of this internship is to further study competitive analysis within the stochastic framework. There are thus two main identifying goals. First, to survey recent progress in the area, and online problems for which stochastic analysis has proven itself useful. Second, and more importantly, to apply this type of analysis to other problems. Of
particular interest are classes of online problems which, in worst case, have very pessimistic worst-case competitive ratios, and which are achieved by very naive algorithms (this includes an entire class of online Steiner tree problems, for instance). The aim is to reduce the big gaps between this pessimistic analysis, and what one should expect in practice.

Qualifications  No prior knowledge of online algorithms is required, but a solid background and general interest in theoretical analysis of algorithms are essential.

References
